COBANet

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1. A conductance based neuron equation

The membrane is a capacitor (C_m) pierced by ion channels in parallel. Conservation of charge (Kirchhoff) balances the capacitive current $C_m\,\mathrm{d}V/\,\mathrm{d}t$ against the total ionic current:

$$C_m \frac{\mathrm{d}V}{\mathrm{d}t} = -\sum_{\text{ion}} I_{\text{ion}} \quad (1)$$

Each channel passes an ohmic current — its conductance $g_{\text{ion}} \geq 0$ times the **driving force** $(V - E_{\text{ion}})$, the distance of V from the reversal potential E_{ion} (where the channel's net current vanishes, set by the Nernst equilibrium):

$$I_{\text{ion}} = g_{\text{ion}}(V - E_{\text{ion}}) \quad (2)$$

Because $g_{\text{ion}} \geq 0$, the current's sign lives entirely in the driving force. Summing a leak (g_L, E_L) and synaptic conductances — excitatory (g_e, E_e), inhibitory (g_i, E_i) — gives the general conductance-based (COBA) neuron:

$$C_m \frac{\mathrm{d}V}{\mathrm{d}t} = -g_L(V - E_L) - g_e(V - E_e) - g_i(V - E_i) \quad (3)$$

2. The COBA model

COBANet specialises (3) to two populations — E driven by excitation and inhibition, I by excitation only:

$$C_m^E \frac{\mathrm{d}V^E}{\mathrm{d}t} = -g_L^E(V^E - E_L) - g_e^E(V^E - E_e) - g_i^E(V^E - E_i) \quad (4)$$

$$C_m^I \frac{\mathrm{d}V^I}{\mathrm{d}t} = -g_L^I(V^I - E_L) - g_e^I(V^I - E_e) \quad (5)$$

A neuron spikes at threshold V_{th} and resets to V_{reset} for a refractory period:

$$s_{t+1} = \mathbb{1}[V \geq V_{\text{th}}], \quad V \leftarrow V_{\text{reset}} \text{ if } s_{t+1} = 1 \text{ or refractory} \quad (6)$$

Each synaptic conductance is an exponential trace driven by presynaptic spikes — each spike adds its full weight as an instantaneous jump, then the conductance decays with the channel time constant; there is no E→E connection:

$$\frac{\mathrm{d}g_e^E}{\mathrm{d}t} = -\frac{g_e^E}{\tau_{\text{AMPA}}} + W_{\text{in}} \sum_k \delta\left(t - t_k^{\text{inp}}\right) \quad (7)$$

$$\frac{\mathrm{d}g_i^E}{\mathrm{d}t} = -\frac{g_i^E}{\tau_{\text{GABA}}} + W_{\text{ie}} \sum_k \delta\left(t - t_k^i\right) \quad (8)$$

$$\frac{\mathrm{d}g_e^I}{\mathrm{d}t} = -\frac{g_e^I}{\tau_{\mathrm{AMPA}}} + W_{\mathrm{ei}} \sum_k \delta(t - t_k^e) \quad (9)$$

(7) is E's excitation from the input W_{in}; (8) its inhibition from I via W_{ie}; (9) the I population's excitation from E via W_{ei}.

3. Discretization

The conductances (7)–(9) and membrane equations (4)–(5) are continuous ODEs. The delta-driven conductances integrate **exactly** over one step: between spikes they decay by $e^{-\Delta t/\tau}$, and any spike landing in the step adds its full weight — the decay-then-add recurrence $g_{t+1} = e^{-\Delta t/\tau} g_t + W s_t$ (with the τ, W and spike train s of each of (7)–(9)). The membrane we integrate by **exponential Euler** — the same algebra for both populations (the I neuron drops g_i).

Collecting on V makes it linear, with total conductance $g_{\mathrm{tot}} = g_L + g_e + g_i$:

$$C_m \frac{\mathrm{d}V}{\mathrm{d}t} = -(g_L + g_e + g_i)V + (g_L E_L + g_e E_e + g_i E_i) \quad (10)$$

Dividing by g_{tot} gives decay-to-steady-state form, naming $\tau_{\mathrm{eff}} = C_m/g_{\mathrm{tot}}$ (shorter than C_m/g_L when synapses are open) and the steady-state voltage V_∞ (the conductance-weighted mean of the reversals):

$$\frac{C_m}{g_{\mathrm{tot}}} \frac{\mathrm{d}V}{\mathrm{d}t} = -\left(V - \frac{g_L E_L + g_e E_e + g_i E_i}{g_{\mathrm{tot}}}\right) \quad (11)$$

A **zero-order hold** freezes the conductances over one step Δt, leaving (11) constant-coefficient with exact solution

$$V_{t+1} = V_\infty + (V_t - V_\infty) e^{-\Delta t/\tau_{\mathrm{eff}}} \quad (12)$$

Per population — I has no g_i, so its g_{tot} and V_∞ drop those terms:

$$g_{\mathrm{tot}}^E = g_L^E + g_e^E + g_i^E, \quad \tau_{\mathrm{eff}}^E = \frac{C_m^E}{g_{\mathrm{tot}}^E}, \quad V_\infty^E = \frac{g_L^E E_L + g_e^E E_e + g_i^E E_i}{g_{\mathrm{tot}}^E} \quad (13)$$

$$g_{\mathrm{tot}}^I = g_L^I + g_e^I, \quad \tau_{\mathrm{eff}}^I = \frac{C_m^I}{g_{\mathrm{tot}}^I}, \quad V_\infty^I = \frac{g_L^I E_L + g_e^I E_e}{g_{\mathrm{tot}}^I} \quad (14)$$

with step (12) for each population $p \in \{E, I\}$: $V_{t+1}^p = V_\infty^p + (V_t^p - V_\infty^p) e^{-\Delta t/\tau_{\mathrm{eff}}^p}$.

Being the *exact* frozen-conductance integral, (12) is **dt-invariant** — N small steps equal one big step, so firing rates and the gamma frequency are physical (Hz) properties, not timestep artifacts (exp044). A forward-Euler step $V_{t+1} = V_t + (\Delta t/C_m) I_{\mathrm{net}(V_t)}$ — not dt-invariant — is kept only as a parity toggle (*COBA_INTEGRATOR*).

Each step runs in fixed order: conductances (7)–(9), then $g_{\mathrm{tot}}, \tau_{\mathrm{eff}}, V_\infty$, then the membrane step (12), then spike + reset (6). **The zero-order hold is this ordering** — the conductances advance once, then stay fixed while the membrane integrates across Δt. E and I advance synchronously, phase-locking the E→I→E gamma cycle to the grid.