A PING rhythmicity metric

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Abstract

A single bounded scalar for how rhythmic a spiking network is: the **lobe–trough contrast** of its spike-time autocorrelation, $(\text{lobe} - \text{trough})/(\text{lobe} + \text{trough}) \in [0, 1)$. Driving each excitatory cell with its own private Poisson input makes the metric rate-invariant by construction; across untrained PING networks it reads 0 along the COBA edges and rises smoothly to 0.98 through the PING interior, a rankable gradient that tracks the gamma-gated collapse of the E firing rate from 95 to 3 Hz.

Methods

The networks here are untrained PING populations driven by external input; the rhythmicity metric is read off the resulting excitatory raster. Write $r(t)$ for the binned population spike count (n bins of width Δt) and ℓ for the lag. The metric is read off in a fixed sequence of steps:

1. **Drive each E cell with its own private Poisson channel.** Every excitatory cell receives an independent homogeneous Poisson spike train at 100 Hz through a one-to-one identity input weight: there is no shared, dense W_{in} projection, so no two cells share an input channel. Private input removes the input-driven spike coincidence that would otherwise inflate the metric at low firing, which is what makes the contrast rate-invariant by construction (Figures 4–5) with no post-hoc correction.
2. **Bin to a population count.** Sum spikes across cells into one count per Δt bin, giving the population trace $r(t)$ of length n.
3. **Raw autocorrelation.** Form the lag product $\sum_t r(t)r(t+\ell)$, which counts spike pairs separated by lag ℓ. All lags are computed at once in $O(n \log n)$ via the Wiener–Khinchin route (zero-pad r, take its FFT, multiply by the conjugate, inverse-transform) rather than the $O(n^2)$ direct sum.
4. **Correct for finite overlap.** Only $n - \ell$ bin-pairs exist at lag ℓ (the last ℓ samples have no partner), so the raw sum tapers toward zero with lag simply from running out of overlap; dividing by that per-lag overlap $n - \ell$ converts it to the *average* product per available pair and flattens the taper.
5. **Set the chance level.** Divide by the mean rate squared $\langle r \rangle^2$ so rate-matched independent firing sits at $A = 1$, and drop the self-paired zero lag. The result is the normalised autocorrelogram

$$A(\ell) = \frac{1}{\langle r \rangle^2} \frac{1}{n - \ell} \sum_t r(t)r(t+\ell).$$

6. **Locate the Mexican hat.** A rhythmic $A(\ell)$ has a "Mexican-hat" profile: a central lobe above 1 (spikes recur a cycle apart) flanked by a dip below 1 where firing is suppressed between volleys. Scanning out from zero lag, take the trough as the first local minimum of a lightly smoothed $A(\ell)$ (it falls near the half-period) and the lobe as the highest point at a shorter lag. Both searches start one bin past zero, so the self-paired zero-lag value dropped in step 5 is excluded from the lobe height: the lobe is read from the first real lag onward, never the trivial self-correlation.

7. **Read the contrast.** The metric is the **lobe–trough contrast**

$$\text{contrast} = \frac{\text{lobe} - \text{trough}}{\text{lobe} + \text{trough}} \in [0, 1),$$

   zero when lobe equals trough (no structure) and approaching 1 as the trough goes silent. It is bounded by construction, with no trough floor needed.

In words: $A(\ell)$ is how much more (or less) likely a spike is to be followed by another one ℓ ms later than under independent firing, with $A = 1$ the chance floor. A central lobe above 1 says spikes *cluster* in volleys; a trough below 1 near the half-period says firing is *suppressed between* them. The contrast is **0** when the spikes carry no such structure (asynchronous) and approaches **1** as sharp volleys separate against near-silence; because it reads the *shape* of $A(\ell)$, it registers a rhythm whether or not its frequency holds still.

Results

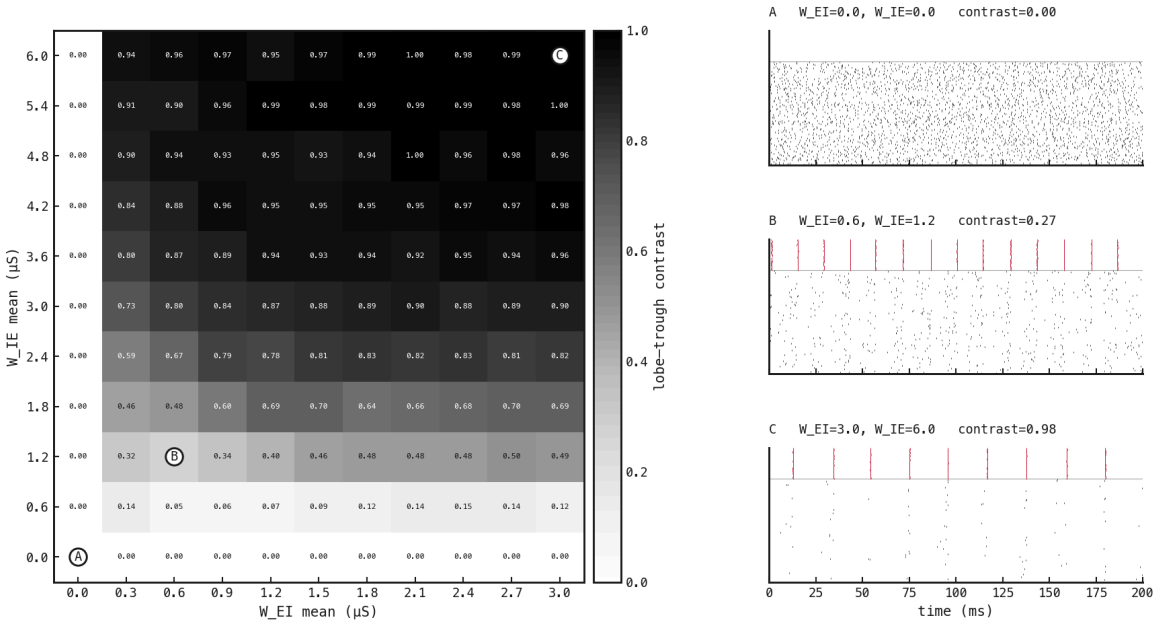

Figure 1: The anchor result: gamma switches on smoothly across the recurrent-weight plane, read as scalar maps over the $W_{EI} \times W_{IE}$ grid (untrained networks, private per-cell Poisson input) with example rasters beneath. **Top**: three per-cell summaries, every cell labelled. **E rate** is high along both zero edges (the loop is broken, E fires at the input-driven ≈95 Hz) and gated down through the interior; **I rate** is silent where $W_{EI} = 0$, runs away along $W_{IE} = 0$ (clipped so the interior is legible), and is controlled once the loop closes; **lobe–trough contrast** (the rhythm scored 0–1) reads exactly 0 along both COBA edges and rises smoothly toward strong coupling, with three points marked along the $W_{IE} = 2W_{EI}$ diagonal. **Bottom**: E/I rasters (E black, I red above) at those points: **A** the fully-off origin ($W_{EI} = W_{IE} = 0$), asynchronous with no I; **B** weak coupling, emerging volleys (contrast 0.27, below the half-way mark); **C** strong coupling, sharp volleys (contrast 0.98). E rate falling, I rate rising, and contrast rising are three readings of the same loop engaging. The full per-cell detail (every raster and autocorrelogram) is in the figures below; the rate-fairness behind the contrast metric is in Figures 4–5.

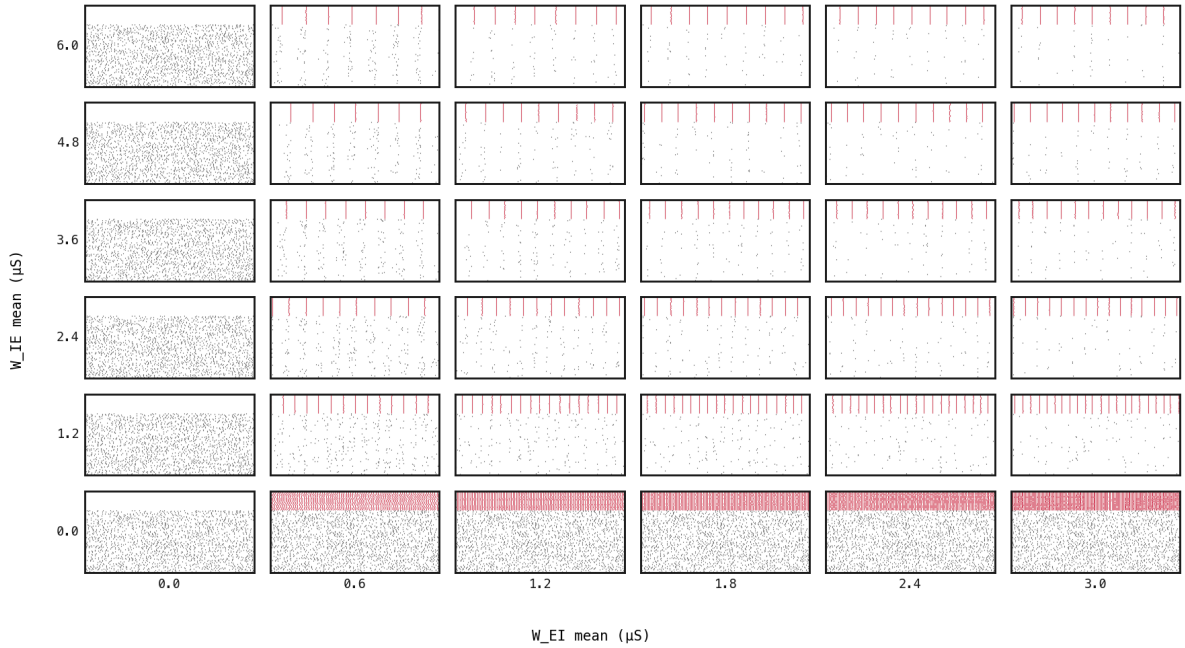

Figure 2: E/I rasters for a 6×6 subset of the grid (every other cell; the heatmaps below use all 121). The two zero edges are the controls: $W_{EI} = 0$ (left column) leaves I silent and E asynchronous; $W_{IE} = 0$ (bottom row) lets I fire but not inhibit E, so both stay dense; neither is rhythmic. The interior shows clear gamma volleys (E black, I red) that sharpen as either weight grows, the rhythm the contrast (Figure 1) scores.

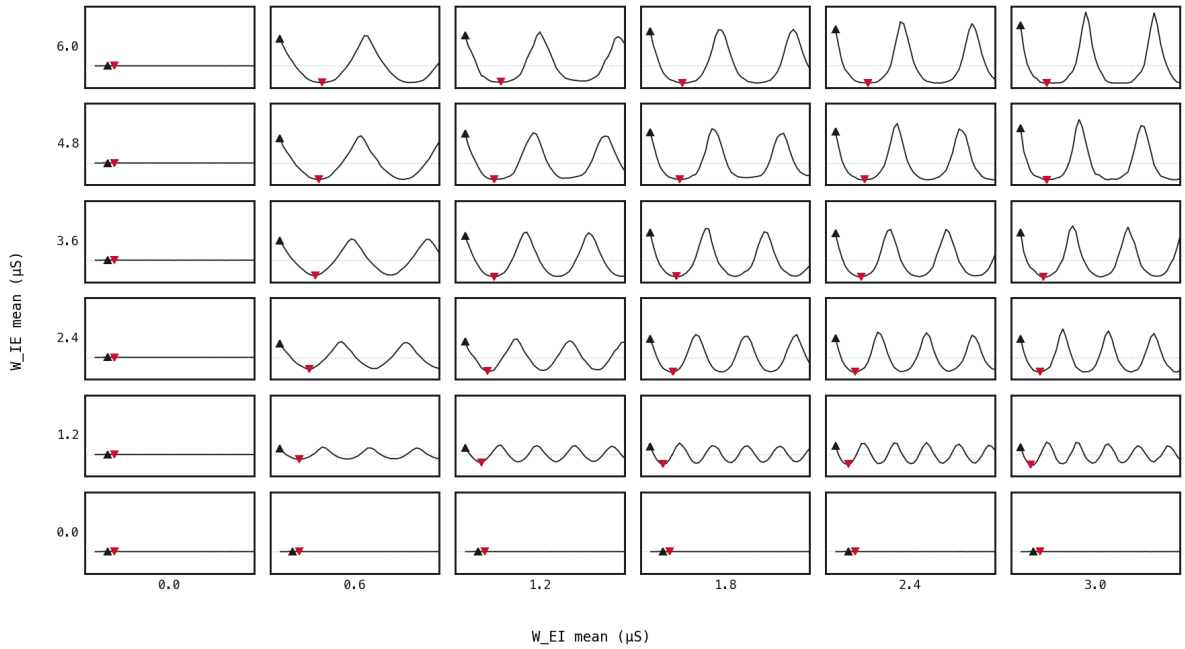

Figure 3: The E-population autocorrelogram $A(\ell)$ for a 6×6 subset of the grid (lag 0–50 ms; dotted line = chance, $A = 1$), with the located lobe (▲) and trough (▼) marked. The two zero edges are flat at 1: asynchronous firing, no structure. Through the interior the Mexican hat emerges: a sharp central lobe at ≈1 ms over a trough near the half-period, with a secondary peak at the full period further out. The contrast in Figure 1 is exactly this lobe-versus-trough read off as one number.

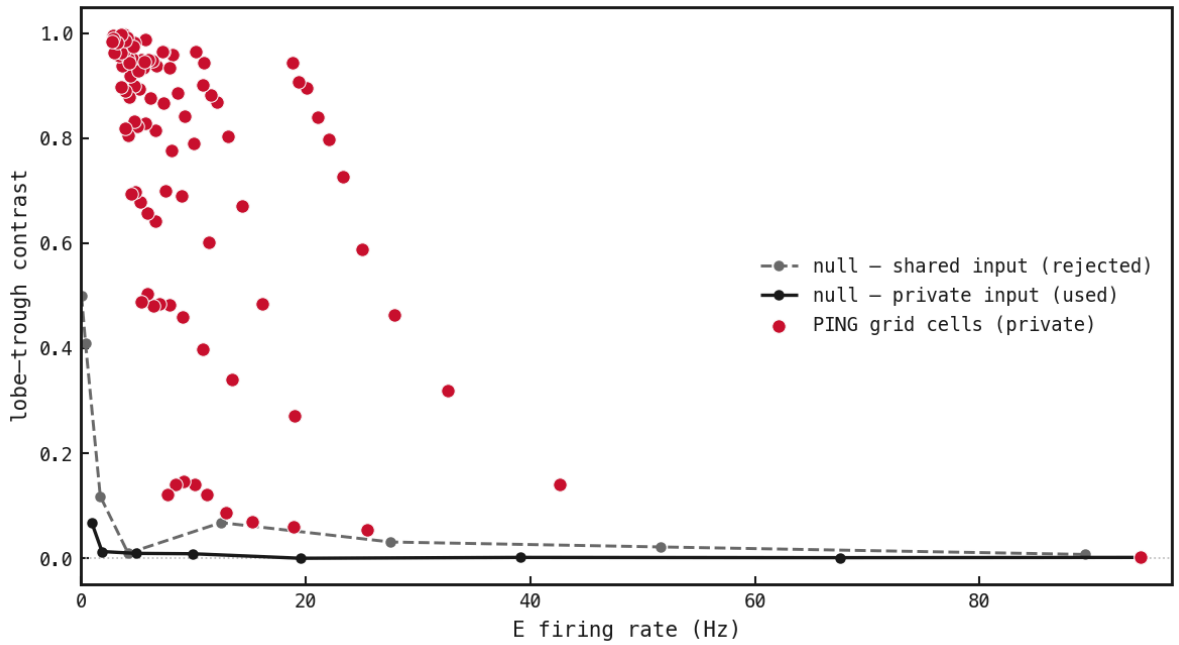

Figure 4: The rate-invariance test that justifies the private-input choice. Each line is a **non-rhythmic null network** (no inhibitory loop, no rhythm at any drive) scanned over input rate; a rate-invariant metric should read ≈0 everywhere. With **private input** (black) it does: flat at ≤0.07 across all firing rates. With **shared input** (grey dashed) it instead **climbs to ≈0.50 as firing thins**: cells sharing input channels fire coincidentally, and the metric reads that as rhythm. The real PING cells (red) sit well above the private-input null, so their contrast is genuine, with no correction needed.

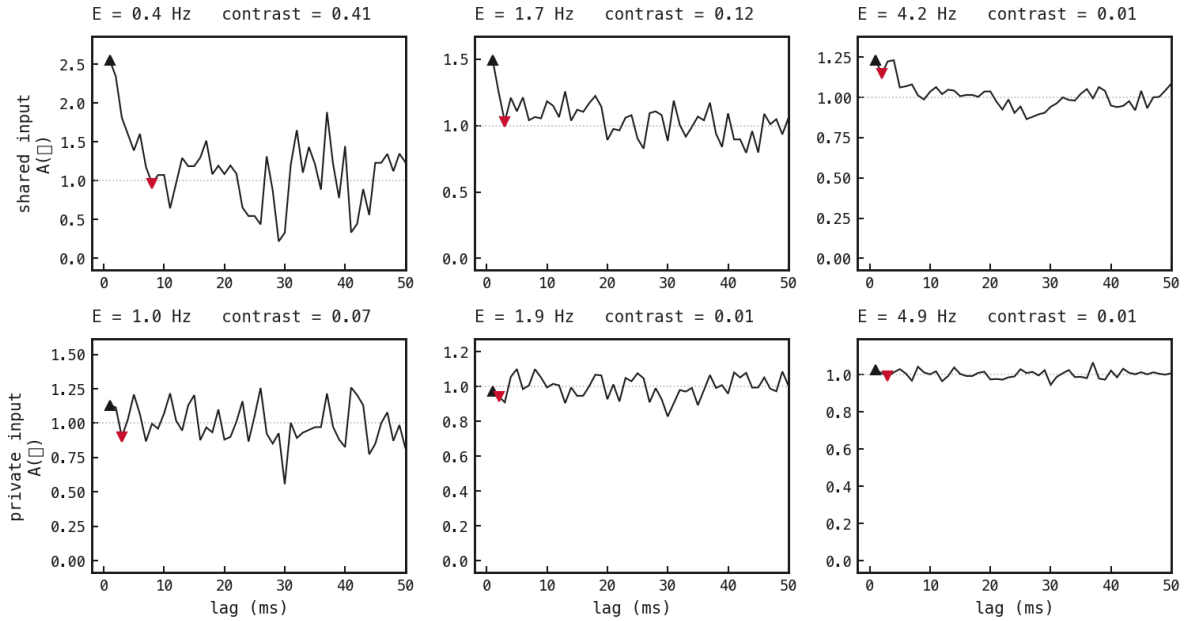

Figure 5: Why shared input fails and private input does not, at matched low firing rates. **Top (shared input):** coincident spikes from shared channels leave a central peak over a shallow dip (a spurious hat with no inhibitory loop behind it), and the metric marks a lobe (▲) and trough (▼) and reports a non-zero contrast. **Bottom (private input):** the same firing rates, but with one channel per cell the central peak is gone; $A(\ell)$ is flat shot-noise around chance and the contrast collapses to ≈0. Same rate, same spike counts; the only difference is whether cells share input.

The onset over its mean-field bifurcation

The turn-on above is *what* the network does; the exp033 4D conductance mean-field is *why*. This final section stacks the two into one manuscript figure: the empirical maps and example rasters directly over the mean-field bifurcation that predicts them. It recomputes the exp033 numerics (Hopf crossing, hysteresis sweep, gamma-vs-τ_{GABA}) and reuses this notebook's own map and raster rendering, so restyling Figure 1 propagates here automatically, and no figures are copied. (This is the anchor that was formerly its own entry.)

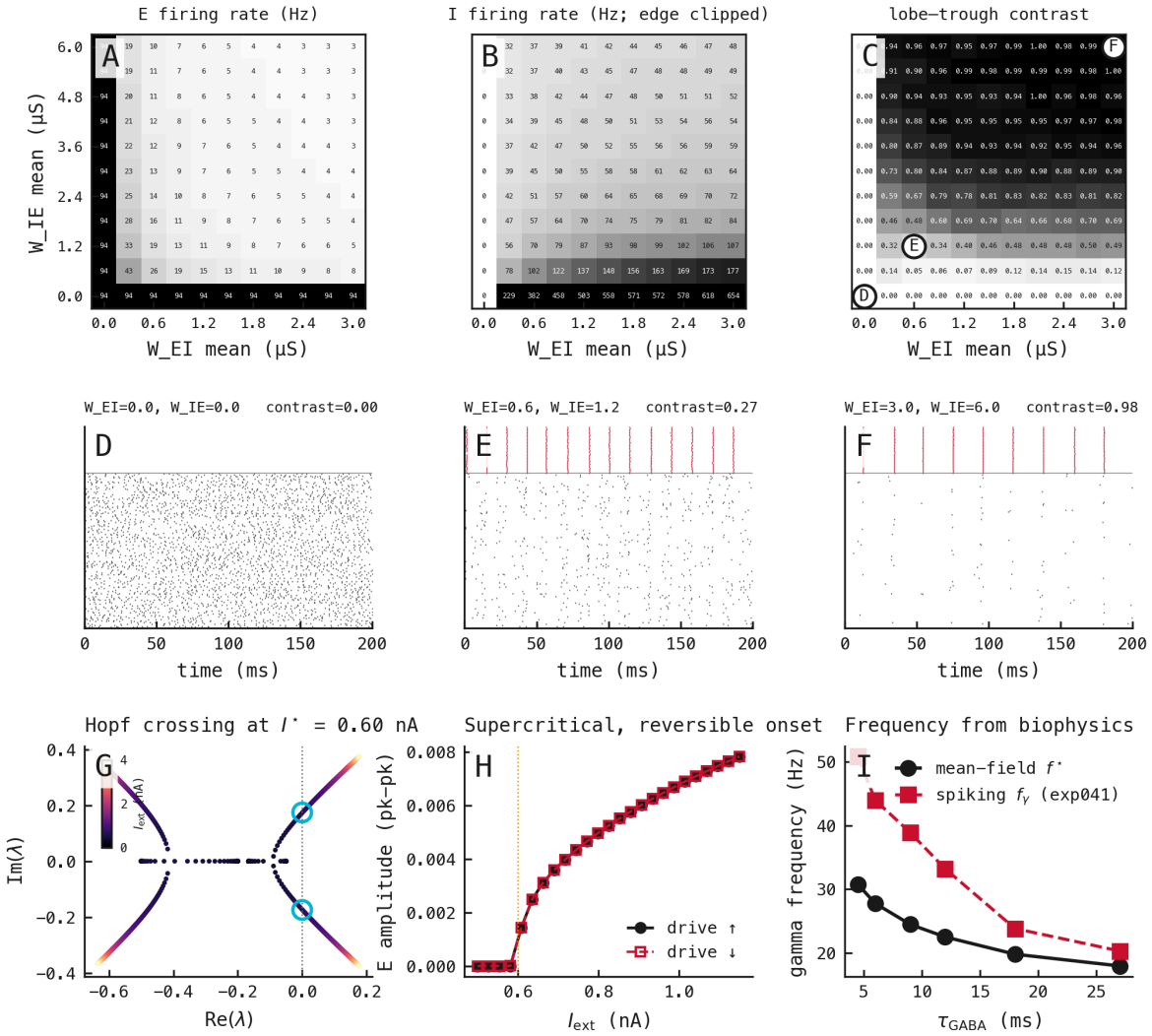

Figure 6: Panels are lettered **A–I** in reading order. **Top (empirics, A–C).** Across the $W_{EI} \times W_{IE}$ plane the E rate falls (**A**), the I rate rises (**B**), and the lobe–trough contrast rises (**C**): three readings of one loop engaging, 0 along both COBA edges and smoothly up toward strong coupling. The contrast map circles three points along the $W_{IE} = 2W_{EI}$ diagonal, shown as rasters **D/E/F**: the fully-off origin (**D**), emerging volleys (**E**), and sharp gamma volleys (**F**). **Bottom (theory, G–I, exp033).** The same onset from a 4D conductance mean-field calibrated from the biophysics: a complex-conjugate eigenvalue pair crosses into the right half-plane at $I^* = 0.60$ nA (a Hopf, **G**), the amplitude rises continuously with coinciding up/down branches (supercritical and reversible, not a hard switch, **H**), and the predicted gamma frequency falls with τ_{GABA} in step with the exp041 spiking measurement (**I**). The smooth empirical turn-on is exactly what a supercritical Hopf predicts.