The Canonical V&S state

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Abstract

Vreeswijk & Sompolinsky predict that a strongly coupled, sparse, inhibition-dominated network falls untuned into a *balanced* state: large E and I currents cancel, and the residual $O(1)$ fluctuations drive irregular (CV ≈ 1), asynchronous, rhythmless firing, *generated* by the deterministic dynamics rather than inherited from the input. We push the conductance-based COBANet into the full four-coupling regime and test three predictions: the balanced signatures at fixed fan-in (Figure 1); their survival as $K \to \infty$ only under strong coupling, synapse $\propto J/\sqrt{K}$ (Figure 2); and that the irregularity is deterministic chaos, by perturbing clones on noise-free input (Figure 3). All three hold: Poisson ISIs, near-zero correlations, a broadband spectrum, a supra-Poisson CV sustained as K grows, and a positive Lyapunov exponent (λ ≈ +28 s^{-1} vs ≈ 0 for a decoupled control). The lone caveat (Figure 2) is a finite-K rate drift.

Methods

1. **Network and the K–J mapping.** COBANet, $N_E = 1024$, $N_I = 256$, $\Delta t = 0.25$ ms, $T = 1000$ ms. **Fixed fan-in** (exact-K, sparsity $s = 0.99$): every cell draws exactly K inputs. **Four recurrent matrices** $W^{EI} = \mathcal{N}(0.6, 0.18)$, $W^{IE} = \mathcal{N}(3.0, 0.9)$, $W^{II} = \mathcal{N}(0.4, 0.12)$, $W^{EE} = \mathcal{N}(0.4, 0.12)$ µS (W^{EE} for completeness: the fixed-K balance sets the rates, not it). **Per-cell independent Poisson drive**, uncorrelated (E: 8 Hz × 2.10 µS, I: 8 Hz × 0.25 µS). The simulator is set by sparsity s and weight w; the theory's fan-in K and coupling J are derived,

$$K = (1 - s)N_{\text{pre}} \quad (1)$$

$$J = \frac{w}{\sqrt{K}} \quad \Longleftrightarrow \quad w = J\sqrt{K} \quad (2)$$

   where:

   - K, the **fan-in**: how many presynaptic cells each neuron receives input from (the size of its "crowd");
   - J, the **coupling**: the $O(1)$ synaptic strength held *fixed* as K grows. The $J/\sqrt{K}$ scaling is the load-bearing V&S choice, keeping the mean recurrent input $O\left(\sqrt{K}\right)$ (large, cancels) while the fluctuation stays $O(1)$ (survives);
   - s, connection **sparsity** (the *–ei-sparsity* flag); $1 - s$ is the probability any given pair is wired;
   - N_{pre}, size of the presynaptic pool a cell draws from (N_E or N_I);
   - w, mean of the recurrent weight matrix in µS (the value passed to *–w-ei* etc.).

Exact-K divides each synapse by its fan-in, so the total a cell receives, $K \cdot (w/K) = w$, is K-independent, giving (2). At $s = 0.99$:

matrix	direction	N_{pre}	w (µS)	K	$J = w/\sqrt{K}$
W^{EI}	E $\to$ I	1024	0.6	≈ 10	≈ 0.19
W^{EE}	E $\to$ E	1024	0.4	≈ 10	≈ 0.13
W^{IE}	I $\to$ E	256	3.0	≈ 3	≈ 1.73
W^{II}	I $\to$ I	256	0.4	≈ 3	≈ 0.23

So $K_E \approx 10$, but the 4× smaller I pool gives only $K_I \approx 3$. V&S needs K_E, K_I the *same order* (4:1 qualifies), but $K_I \approx 3$ is marginal: at equal fan-in ($K_I \approx 10$) the state stays balanced and asynchronous, the CV merely relaxing from 1.1–1.2 to ≈ 1.0, so the supra-Poisson burstiness is a lumpy-K_I shot-noise effect, not a balance failure.

2. **Why it is irregular.** Each cell sums $\approx K$ synapses of strength $\propto J/\sqrt{K}$, so the mean recurrent input is $O\big(\sqrt{K}\big)$ but its fluctuation is $O(1)$. A moderate rate makes the large opposing E and I means **cancel to leading order**, parking the drive near threshold; the residual $O(1)$ fluctuations carry each cell over threshold at random times (Poisson, CV $\to$ 1), and since each cell hears its own uncorrelated input, the population is asynchronous. These fluctuations are network-*generated*: they survive $K \to \infty$ only under $J/\sqrt{K}$ scaling (Step 4) and persist under noise-free input (Step 5).

3. **Diagnostics.** From post-burn-in spikes: per-neuron ISI CV, the Welch spectrum of the population-mean E trace, and the mean pairwise cross-correlogram over 100 random E pairs.

4. **The $J/\sqrt{K}$ coupling test.** Strong coupling means synapse $\propto J/\sqrt{K}$, keeping an $O(1)$ recurrent fluctuation as $K \to \infty$; holding w fixed instead gives synapse $\propto 1/K$, the weak (mean-field) limit, fluctuation $\propto 1/\sqrt{K} \to 0$. At fixed $N_E = 1024$ we sweep $K = 10 \to 160$ under both: **strong** (weights $\propto \sqrt{K}$, drive folded into the balance, so rate $\propto K$ and $g \propto 1/\sqrt{K}$) and **weak** (all fixed), reading off whether the irregularity survives. (3 s trials, since the strong rate drifts low at high K and the CV needs enough ISIs.)

5. **Direct Lyapunov exponent (frozen input).** Replace the Poisson drive with a **quenched DC conductance** (a frozen per-cell offset, no per-timestep fluctuation), run two clones on *identical* input, kick every voltage by ε at $t = 0$, and track the **voltage distance** between the copies,

$$\|\Delta V(t)\| = \sqrt{\sum_{i=1}^{N_E} \left(V_i^{\text{clean}}(t) - V_i^{\text{pert}}(t)\right)^2} \quad (3)$$

the Euclidean distance between the two copies' N_E E-cell membrane-voltage vectors (V_i^{clean} unperturbed, V_i^{pert} kicked). With noise-free input any divergence is network-generated. We compare the **balanced** four-coupling net against a **decoupled** control: the same COBA E and I cells under the same DC drive, but with all four recurrent matrices set to ≈ 0, so no cell receives synaptic input from any other. Each decoupled cell is then an independent DC-driven integrator (a clock), and a perturbation has nowhere to spread, fixing the $\lambda \approx 0$ baseline. (It is *not* a feedforward network: there is no input-layer pathway, just the same architecture with every recurrent wire cut.) Spiking dynamics contract between spikes and expand only at spike-flips, so ε must flip spikes ($\varepsilon = 0.1$ mV; smaller just contracts $\rightarrow$ spurious $\lambda < 0$). The slope of $\log\|\Delta V\|$ in the post-flip, pre-saturation window is the largest Lyapunov exponent (five seeds). It is an *initial-growth* estimate ($\|\Delta V\|$ saturates at the attractor size), so the sign, not the magnitude, is the result.

Results

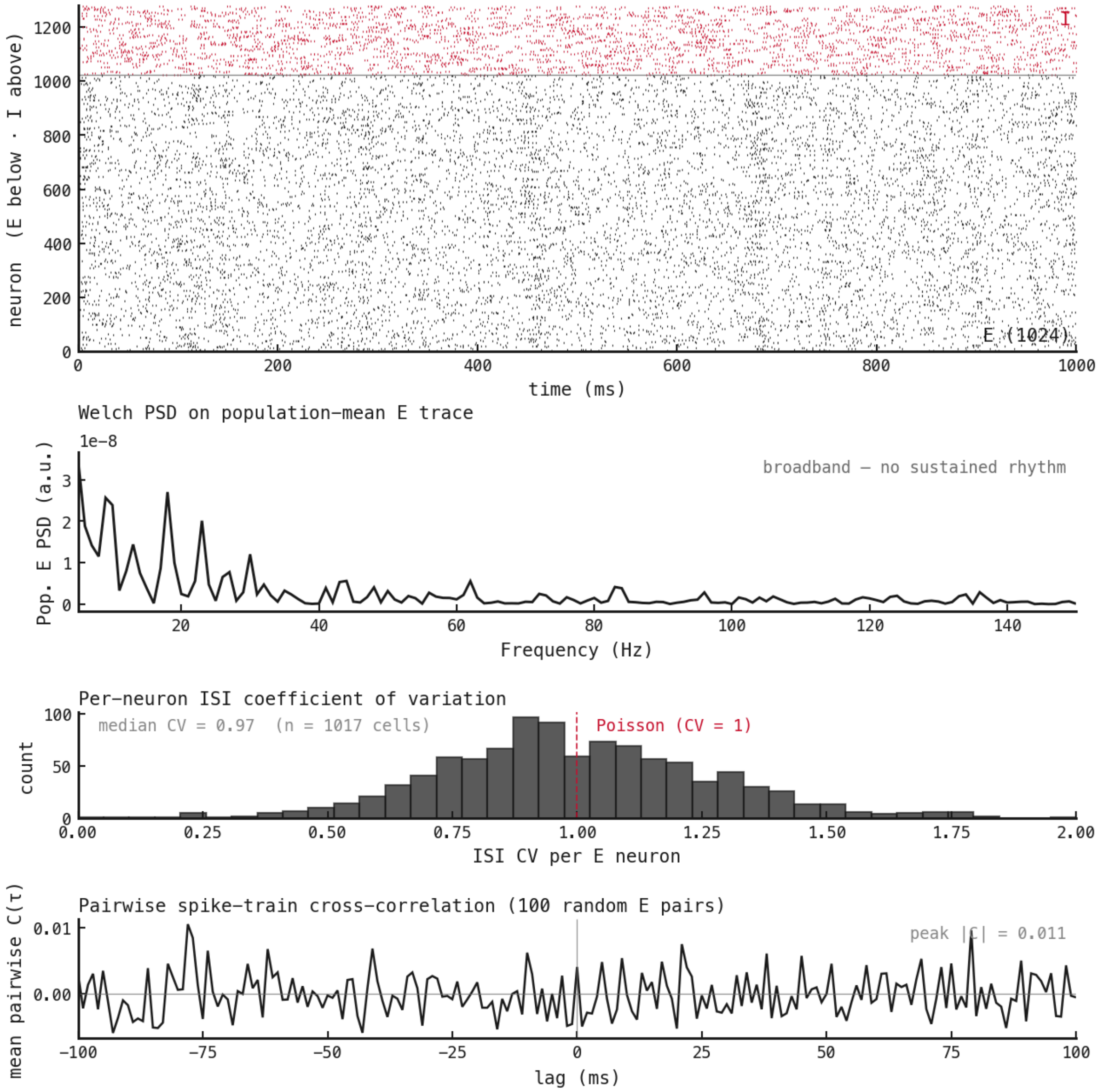

Figure 1: Four diagnostics of the four-coupling state (E black, I red above the divider; ⟨r_E⟩ ≈ 17.5, ⟨r_I⟩ ≈ 25.4 Hz). **What we expect.** Irregular (ISI CV ≈ 1; a constant-current cell sits at 0), asynchronous (near-zero pairwise correlation), rhythmless (broadband, no peak). **What we see.** *Raster:* scattered, no bands. *PSD:* broadband, no sustained rhythm; the single-trial spectral max wanders seed to seed (5–90 Hz across six seeds), so it is not a peak, but a *weak* gamma-band E–I resonance does survive seed-averaging (≈ 2× the floor near 40 Hz, the same E↔I loop that sharpens into PING gamma, here heavily damped). *ISI CV:* median 1.11 (E), 1.20 (I). *Cross-correlogram:* flat, peak |C(τ)| ≈ 0.01 despite heavy shared input, the active decorrelation of Renart et al. (2010). **Do they align?** Yes on all three: the COBANet enters the balanced state untuned.

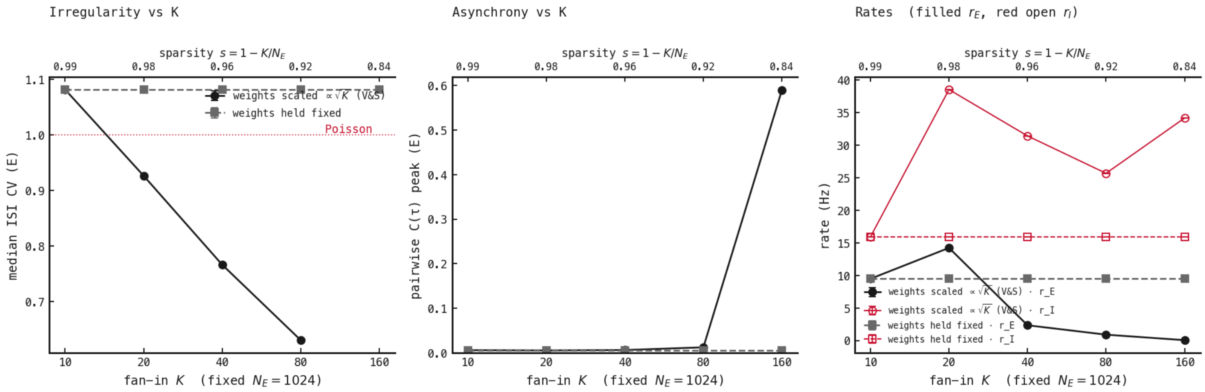

Figure 2: Fan-in $K = 10 \to 160$ at fixed $N_E = 1024$, equivalently sparsity $s = 1 - K/N_E$ from 0.99 to 0.84 (top axis), with recurrent weights scaled $\propto \sqrt{K}$ (black, the V&S $J/\sqrt{K}$ rule) versus held fixed (grey); r_I shown in red. ±1 SD over three seeds, 3 s trials. **What we expect.** Strong coupling keeps the $O(1)$ fluctuations, so irregularity *survives* as K grows; weak coupling loses them ($\propto 1/\sqrt{K}$), so cells *regularise*. Asynchrony holds in both. **What we see.** *Irregularity:* strong stays supra-Poisson (CV 1.2–1.3, easing to 1.11 at $K = 160$); weak decays to ≈ 1.0, the Poisson floor. *Asynchrony:* both ≈ 0.006 throughout. *Rates:* strong r_E drifts 17.5 → 4.9 Hz (the $O\left(1/\sqrt{K}\right)$ finite-K correction); weak holds ≈ 17 Hz. **Do they align?** Yes: only $J/\sqrt{K}$ keeps the recurrent irregularity alive as K grows, and the gap is the signature. The one mismatch is the strong-coupling rate drift (Next steps); whether the survival is *self-generated* is settled by Figure 3.

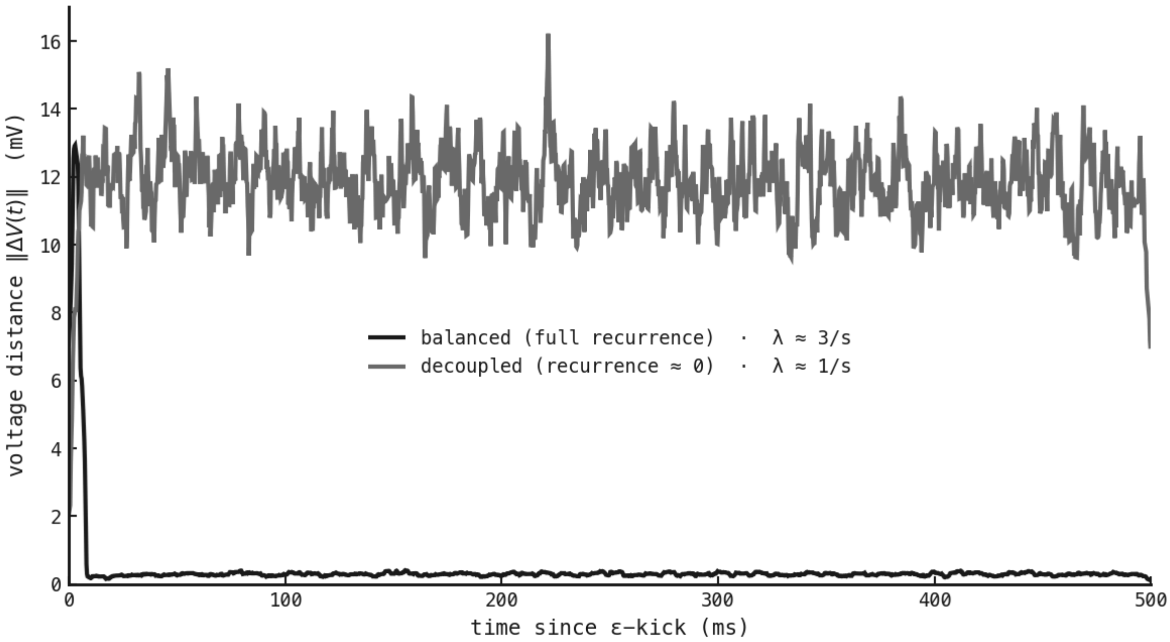

Figure 3: Two clones on *identical* quenched-DC input, the second kicked by ε = 0.1 mV at $t = 0$; voltage distance $\|\Delta V(t)\|$, seed-mean (bold) with a ±1 SD band over five seeds, lightly smoothed. **What we expect.** If chaotic, the kick is *amplified* ($\|\Delta V\|$ grows exponentially, λ > 0), and with noise-free input that can only be network-generated. The decoupled control should not grow (λ ≤ 0). **What we see.** *Balanced (black):* $\|\Delta V\|$ rises steeply (fitted rate λ ≈ +28 s^{-1}) then saturates at the attractor size (≈ 250 mV). *Decoupled (grey):* neither amplified nor forgotten, a fixed phase offset, λ ≈ 0. **Do they align?** Yes: λ > 0 for the balanced net, ≈ 0 for the control, deterministic chaos, self-generated. Caveat: an *initial-growth* estimate (saturation caps the window), so the sign, not the magnitude, is the result; a renormalised Benettin scheme would pin exact λ (Next steps).

Next steps

Figures 1–3 confirm the balanced signatures, their $J/\sqrt{K}$-only survival, and a positive Lyapunov exponent: the state is self-generated and deterministically chaotic. Two refinements remain.

1. **Hold the rate as K grows.** Strong-coupling r_E drifts 17.5 → 4.9 Hz (the $O\left(1/\sqrt{K}\right)$ finite-K correction). A DC offset to E alone *fails*: it pushes E off the balanced point and inflates CV to 1.4–1.7. Honest pinning must co-adjust the E *and* I drives along the balanced manifold (solve the 2D balance for a target (r_E, r_I)), giving a clean constant-rate CV comparison.
2. **Pin down the asymptotic λ.** Figure 3 gives the sign; the magnitude is an initial-growth estimate (ε-dependent, saturation-capped). The exact exponent needs a *renormalised* Benettin scheme (lockstep clones, periodic rescaling of ΔV, averaging log-growth over a long trajectory), which also yields the full spectrum and the attractor dimension.